

Integration of Products "By Parts"

We know that from differential Calculus if u, v_1 are two differentiable functions of x

$$\text{then } \frac{d}{dx}(uv_1) = u \frac{dv_1}{dx} + v_1 \frac{du}{dx}$$

Integrating both sides.

$$\Rightarrow uv_1 = \int \left(u \frac{dv_1}{dx} \right) \cdot dx + \int \left(v_1 \cdot \frac{du}{dx} \right) \cdot dx$$

$$\Rightarrow uv_1 - \int \left(v_1 \cdot \frac{du}{dx} \right) \cdot dx = \int \left(u \cdot \frac{dv_1}{dx} \right) \cdot dx$$

$$\text{let } \frac{dv_1}{dx} = v \Rightarrow v_1 = \int v \cdot dx$$

$$\Rightarrow \int (uv) \cdot dx = u \int v \cdot dx - \int \left\{ \left[\int v \cdot dx \right] \cdot \frac{du}{dx} \right\} \cdot dx$$

Thus $\Rightarrow$ Integral of product of two functions

= 1st Function (unchanged) $\times$ Integral of 2nd

— Integral of { diff. Coeff of 1st $\times$ integral of 2nd }

Formulas:-

$$(i) \int x^n \cdot dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$(ii) \int \frac{1}{x} \cdot dx = \log x$$

$$(iii) \int e^x \cdot dx = e^x$$

$$(iv) \int a^x \cdot dx = \frac{a^x}{\log a}$$

$$(v) \int \sin x \cdot dx = -\cos x$$

$$(vi) \int \cos x \cdot dx = \sin x$$

$$(vii) \int \tan x \cdot dx = \log \sec x \text{ or } -\log \cos x.$$

$$(viii) \int \cot x \cdot dx = \log \sin x.$$

$$(ix) \int \sec x \cdot dx = \log(\sec x + \tan x) = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$(x) \int \operatorname{cosec} x \cdot dx = \log(\operatorname{cosec} x - \cot x)$$

$$= \log \tan \frac{x}{2}.$$

$$(xi) \int \frac{1}{a^2+x^2} \cdot dx = \frac{1}{a} \tan^{-1} x/a$$

$$(xii) \int \frac{1}{\sqrt{a^2-x^2}} \cdot dx = \sin^{-1} x/a$$

$$(xiii) \int \frac{1}{\sqrt{x^2-a^2}} \cdot dx = \cos^{-1} x/a$$

$$(xiv) \int \sqrt{a^2-x^2} \cdot dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$(xv) \int \sqrt{a^2+x^2} \cdot dx = \frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

$$(xvi) \int \sqrt{x^2-a^2} \cdot dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$$

$$(xvii) \int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1} x/a$$

$$(xviii) \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right)$$

$$(xix) \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right)$$

$$(xx) \int e^{ax} \sin bx \cdot dx$$

$$= \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$$

$$(xxi) \int e^{ax} \cos bx \cdot dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$$

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